Math 20550 - Summer 2016
Motion in Space and Functions of Several Variables
June 21, 2016

Problem 1. *Suppose a particle is moving with position function* $\mathbf{r}(t) = \langle -\frac{1}{2}t^2, t \rangle$.
(a) *Find the velocity of the particle.*
(b) *Find the acceleration of the particle.*
(c) *Sketch the trajectory of the particle and draw the velocity and acceleration vectors at* $t = 2$.

Problem 2. *Suppose a particle of mass* 3 *is moving with position function* $\mathbf{r}(t) = \langle 2\cos t, 3t, 2\sin t \rangle$.
(a) *Find the velocity of the particle.*
(b) *Find the acceleration of the particle.*
(c) *Find the speed of the particle as a function of time.*
(d) *Find the force the particle experiences.*
(e) *Find the momentum of the particle. (Recall that momentum is given by* $\mathbf{p}(t) = m\mathbf{v}(t)$.)
(f) *Find the angular momentum of the particle. (Recall that angular momentum is given by* $\mathbf{L}(t) = \mathbf{r}(t) \times \mathbf{p}(t)$.)
(g) *Find the torque the particle experiences. (Recall that torque is given by* $\tau(t) = \mathbf{r}(t) \times \mathbf{F}(t)$.)

Problem 3. *A particle is moving with acceleration function* $\mathbf{a}(t) = \mathbf{i} + 2\mathbf{k}$. *Find the position function of the particle if its initial velocity is* $\mathbf{v}(0) = \mathbf{i}$ *and* $\mathbf{r}(0) = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$.

Problem 4. *A gun has muzzle speed* $150m/s$. *What two angles of elevation can be used to hit a target* $800m$ *away? (Assume the gun is firing from ground level.)*

Problem 5. *Find the normal and tangential components of acceleration for a particle moving with position*
$$\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}.$$

Problem 6. *Find the normal and tangential components of acceleration for a particle moving with position*
$$\mathbf{r}(t) = \left\langle e^t, \sqrt{2}t, e^{-t} \right\rangle.$$

Problem 7. *Find and sketch the domain of*
$$f(x, y) = \sqrt{36 - 9x^2 - 4y^2}.$$

Problem 8. *Draw a contour plot for the function*
$$f(x, y) = \sqrt{36 - 9x^2 - 4y^2}.$$

Problem 9. *Plot the graph of the function*
$$f(x, y) = \sqrt{36 - 9x^2 - 4y^2}.$$

Problem 10. *Describe what level surfaces of* $f(x, y, z) = x^2 + y^2 - z^2$ *look like.*